Fluency

July 23-25, 2013

Math & Science Collaborative

Fluency

- What is fluency?
- Take a few minutes and write down a few thoughts about what you think of when you hear “fluency.”

Article Discussion

- **Read: "Fluency: Simply Fast and Accurate? I Think Not!"** by NCTM Past President Linda M. Gojak
- How do the ideas in the article resonate with your ideas?
- What is the same? What is different?
- What stood out for you?

Precursors to Fluency

- **The sequence of number names, both starting at 1 and not starting at 1**
- **How to count a set, keeping track of the items they counted**
- **Understanding relationships of more, less , and same**
- **Skip counting starting from 1 and from other numbers**
- **Cardinality**
- **Conservation**
- **1-to-1 correspondence**
- **Making tens – link to understanding place value**
- **Subitizing**
- **Decomposing and composing numbers**
- **Understanding part-part-whole**
- **Number sense**

Which precursors to fluency do you see in this student work?

- Read the two scenarios of student work and decide which, if any, precursors to fluency are evident in their solutions.

Number Sense:

- **understanding the relationships between and among numbers,**
- **having the ability to think flexibly about numbers and to break numbers apart and put them back together,**
- **being familiar with the properties of single digit numbers and using this information to calculate efficiently using larger numbers,**
- **having the ability to manipulate numbers in their head, and**
- **having effective ways to estimate.**

M. Burns

Number Sense:

NCTM PSSM 2000 says on p. 32:

"number sense – the ability to decompose numbers naturally, use particular numbers like 100 or ½ as referents, use the relationship among arithmetic operations to solve problems, understand the base 10 number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers (Sowder 1992)"

Students with number sense…

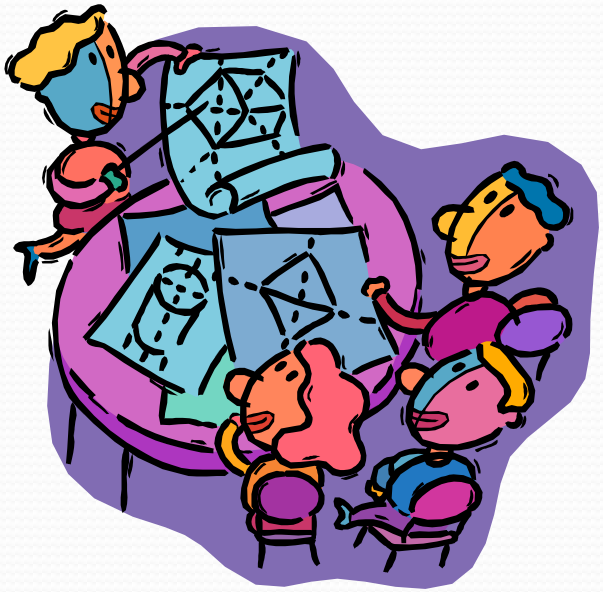
- Have a sense of what numbers mean, can visualize how much 100 is or can see what 1/4 looks like
- Ability to look at world in terms of quantity and numbers, understand when 100 is a lot and when it's not much at all
- Ability to make comparisons among quantities, know 300 is 400 away from 700 or know there is a bigger difference between 50 and 150 than between 1000 and 1050 (use landmark numbers and mental number line)
- Flexibility, automaticity, and fluidity with numbers, use 5 and 10 structures of numbers, place value understandings, and relationships among numbers

Students with number sense…

- Ability to perform mental math, can solve 20+35 by breaking numbers apart
- Flexibility with problems, can solve problem more than one way
- Automatic use of math information, readily use what they know about numbers and number relationships to solve problem
- Ability to determine reasonableness of an answer
- Ability to decide on strategy based on numbers in problem, solve 100—95 by knowing 95 is 5 away from 100 rather than using standard algorithm

Number Sense and Fluency

- Discuss with your table group. Does number sense affect fluency? Why or why not?



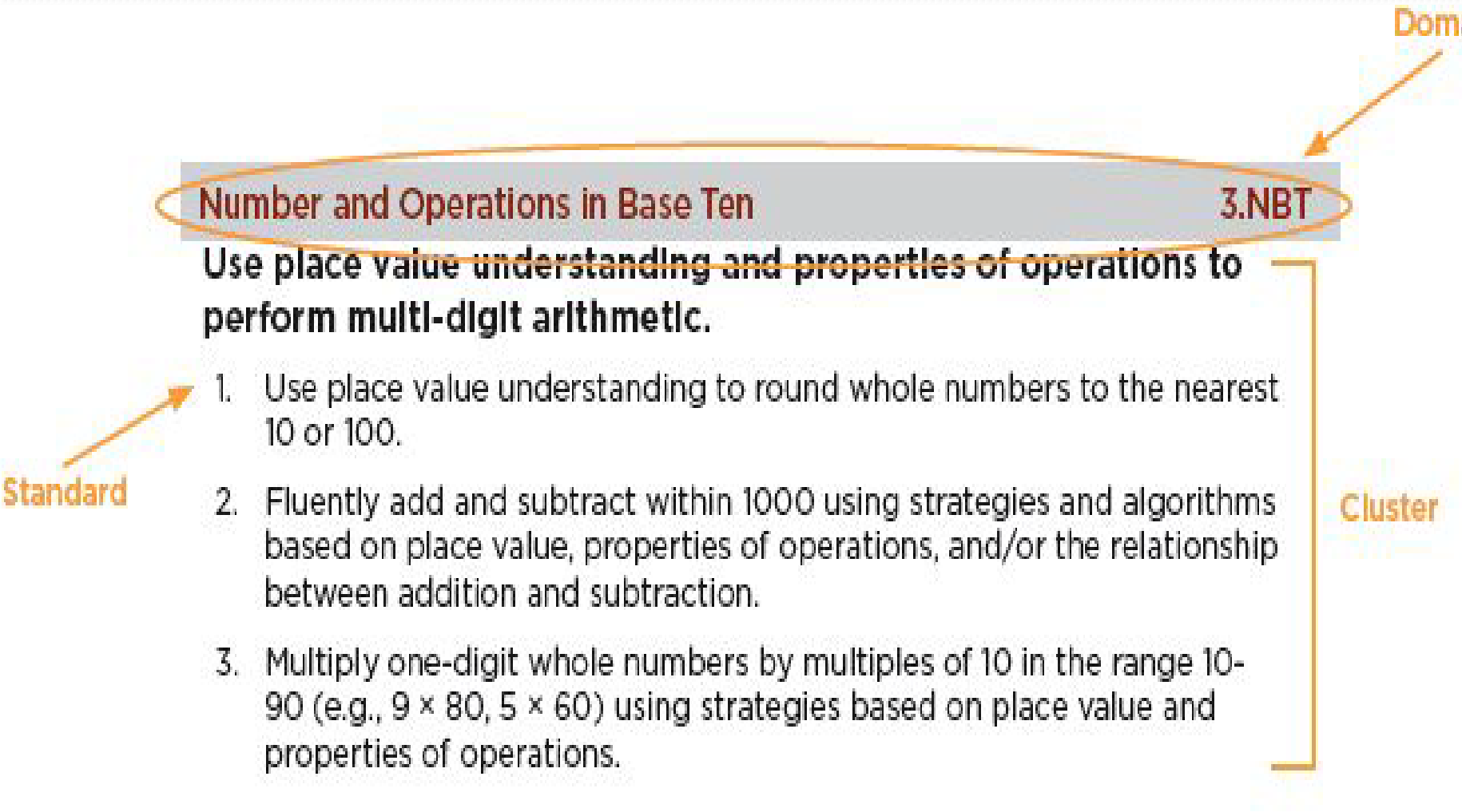
Read and Discuss

- **Read: "Why Children Have Difficulties Mastering the Basic Number Combinations and how to Help Them."** from TCM, August 2006
- What are some reasons why students have difficulty mastering basic combinations?
- What new ideas did you discuss in your group about the relationship between number sense and fluency after reading the article?

"Mastery of a basic fact means that a student can give a quick response (in about 3 seconds) without resorting to inefficient means, such as counting by ones"

(Van de Walle, Karp, Bay-Williams, 2013, page 171).

Common Core State Standards



Common Core State Standards

- Content Standards Jigsaw Activity
  - Count off by 4s
  - All number 1s will be in same expert group
    - Counting and Cardinality (CC) for K
    - Number and Operations in Base Ten (NB) for grades K and 1
  - All number 2s will be in same expert group
    - NB for grade 2
    - Operations and Algebraic Thinking (OA) for grades K and 1
  - All number 3s will be in same expert group
    - NB for grades 3 and 4
    - OA for grades 2 and 3
  - All number 4s will be in same expert group
    - NB for grade 5
    - OA for grades 4 and 5

Common Core State Standards

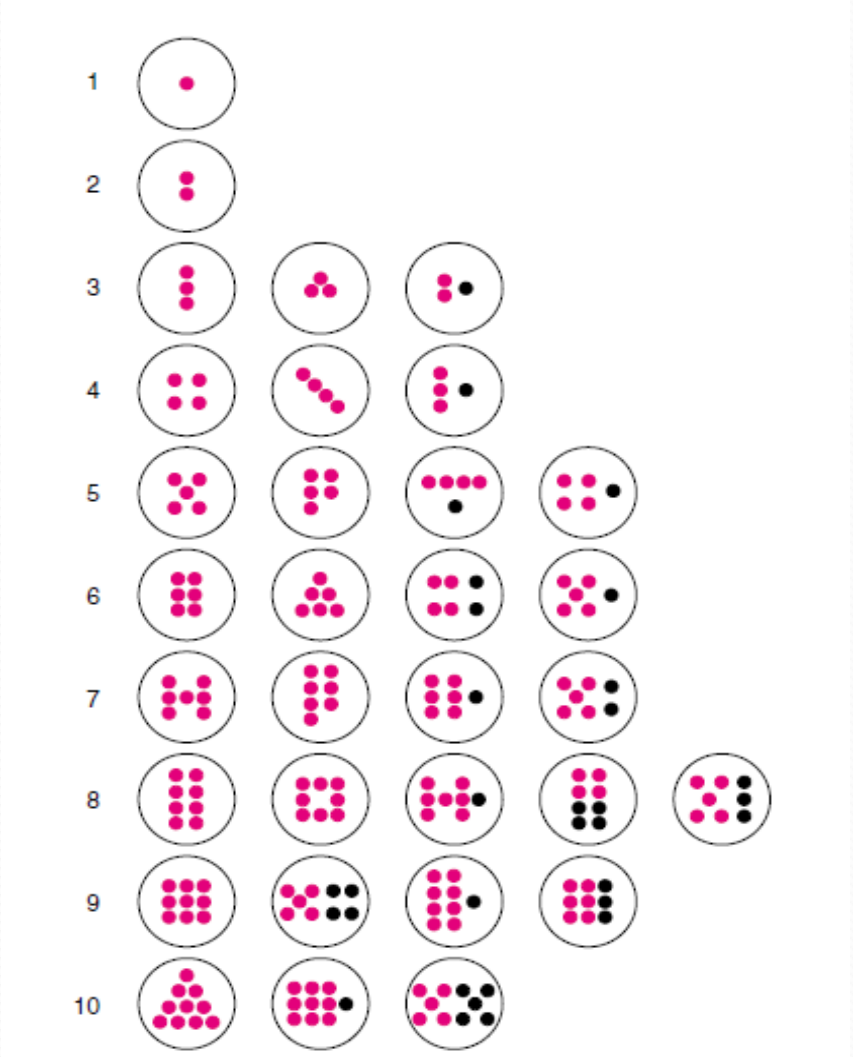
- Read the content standards for
  - Big Take-Aways or "Ahas"
  - Evidence of any precursors to fluency and / or fluency expectations
  - Evidence of how the standards view fluency – from a *conventional* point of view or from a *number sense* point of view.
    - Be able to cite your evidence in the article and / or in the standards themselves.

Number Relationships

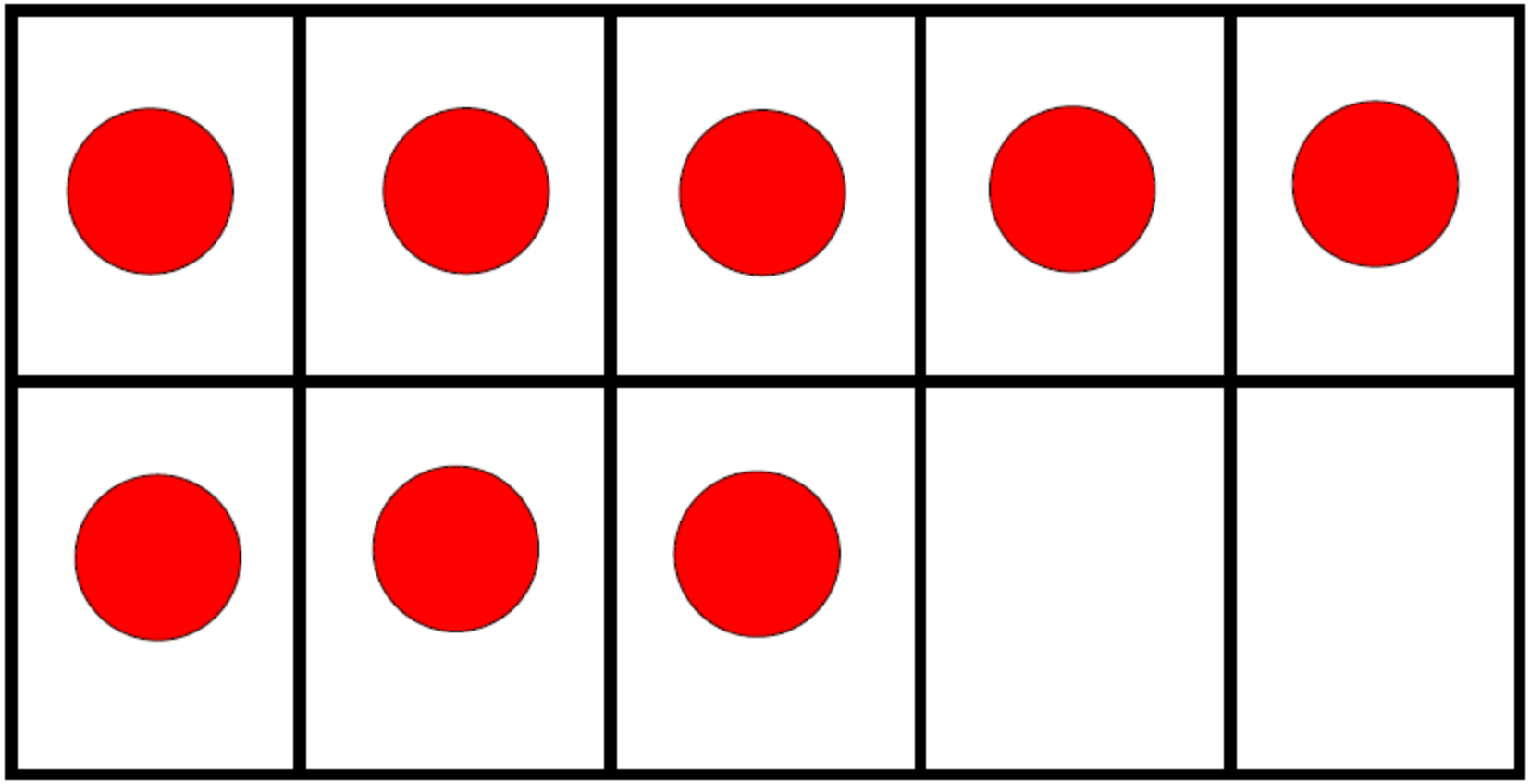
- **Spatial relationships**
- One more/Two more/One less/Two less
- Anchors to 5 and 10
- Part-Part-Whole

Spatial Relationships

- Subitizing – recognizing a quantity without having to count can aid in counting on
  - Dot arrangements – dice, dominoes



Spatial Relationships



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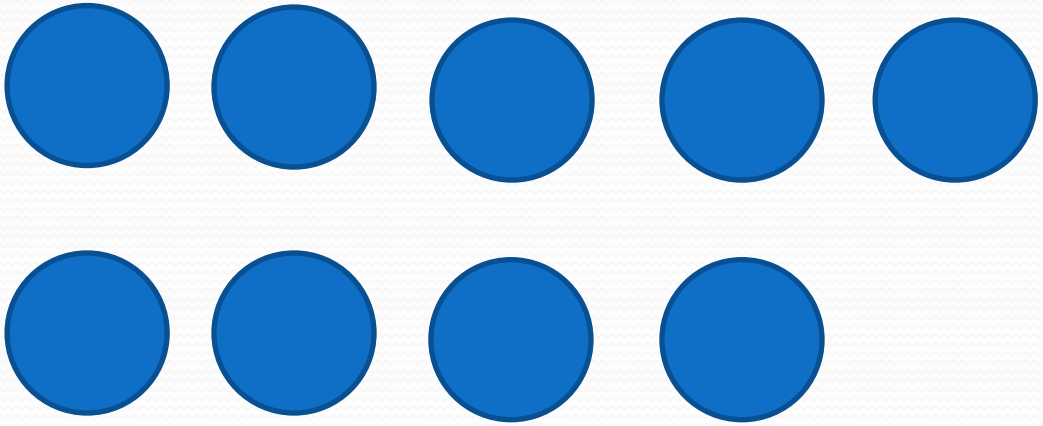

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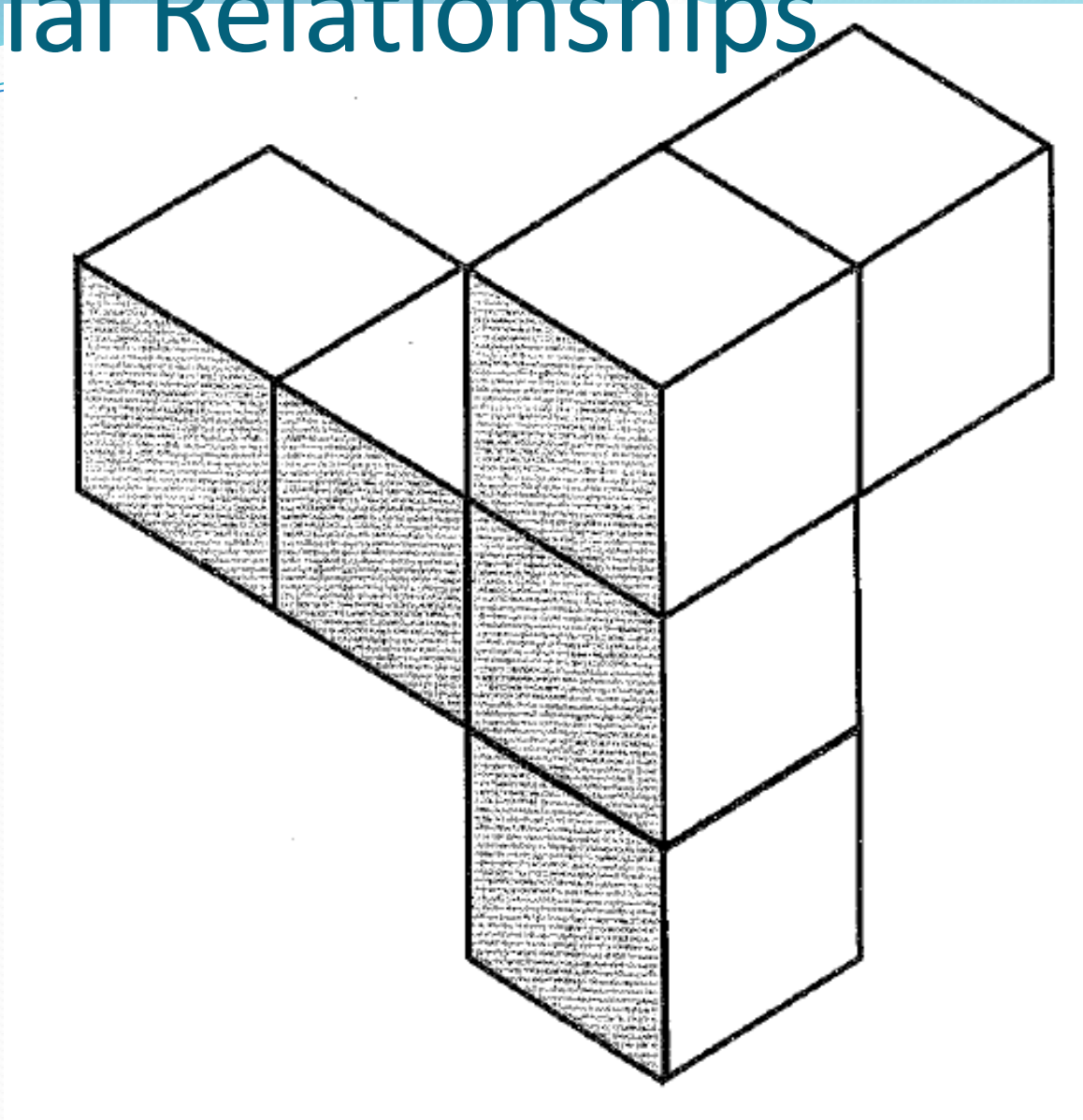

Subitizing: Fundamental Skill in Development of Number

- **Read the article "Subitizing: What is it? Why teach it?"**
- Discuss with your group: Why is subitizing such an important skill?
- Are there any connections between subitizing and the content standards? If so, what are they?

Visualizing on the Ten Frame

- Why is it helpful for students to discuss the mental images they formed?
- Ms. Latimer takes multiple student answers to the same question. What effect does this have?
- How could this lesson be connected with addition and subtraction?

Standards for Mathematical Practice

- Read
  - Elementary Elaborations of Standards for Math Practice 2, 3, 7, and 8 only.
  - Highlight the key ideas from your perspective
- Turn and Talk
  - About what you highlighted as key ideas, then
- As a table group, discuss:
  - To what extent do you think these practices are embedded in the daily work of teachers and students?

Standards for Mathematical Practice

- Are Ms. Latimer's students engaged in Math Practice 7? If so, in what way?
- Are her students engaged in any of the other Math Practices? If so, how do you know?

Video: 8 + 6

- What are examples of how the ten-frame model supports student strategies for 8 + 6?
- How does the ten-frame encourage students' understanding of 10 as an important number in computation?
- What seems to be a core strategy used throughout this number talk?
- 8 + 6 is considered to be a basic addition fact. What role can number talks play in helping students build basic fact knowledge?

Number Relationships

- Spatial relationships
- **One more / Two more / One less / Two less**
- Anchors to 5 and 10
- Part-Part-Whole

One and Two More, One and Two Less

- These relationships involve counting on one or two and counting back one or two.
- They also involve knowing that 7 is one more than 6 and two less than 9.

Number Relationships

- Spatial relationships
- One more/Two more/One less/Two less
- **Anchors to 5 and 10**
- Part-Part-Whole

Anchoring Numbers to 5 and 10

- Since 10 plays a large role in our numeration system and two fives make a ten, it is very useful to develop relationships for the numbers 1 to 10 to the anchors of 5 and 10.
  - This helps in thinking about various combinations of numbers. For example, knowing that 8 is 5 and 3 and is 2 away from 10 can play a role in 5 + 3, 8 + 6, 8 – 2, 8 – 3, 8 – 4, 13 – 8.
- Ten frame is most important model for this relationship.

Popsicle Stick Math

- How does using popsicle sticks help students learn about addition and subtraction?
- Why does Ms. Saul do this activity one-on-one rather than with a large group?
- How is this activity a telling assessment of student understanding?
- Could this activity also be done using a ten-frame? What advantages disadvantages world there be to using ten-frames rather than popsicle sticks?

Number Relationships

- Spatial relationships
- One more/Two more/One less/Two less
- Anchors to 5 and 10
- **Part-Part-Whole**

Part-Part-Whole Relationships

- Conceptualizing a number as being made up of two or more parts is the most important relationships that can be developed among numbers.
  - Focus on quantity in terms of its parts

Mental Math

- Encourages students to build on number relationships to solve problems instead of memorized procedures
- Using number relationships helps students develop *efficient*, *flexible* strategies with *accuracy*
- Causes students to be efficient to avoid holding numerous quantities in their heads
- Strengthens students' understanding of place value

Fluency demands…

- An understanding of the meaning of operations and their relationship to each other.
- The knowledge of a large repertoire of number relationships. (The “patterns” of our number system.)

Fluency demands…

- A thorough understanding of the base-ten number system, how numbers are structured in this system, and how this system behaves in different operations.
- Knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems.

Involves three components.

Fluency

- *Efficiency* implies that the student does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.
- *Accuracy* depends on several aspects of the problem-solving process, among them, careful recording, the knowledge of basic number combinations and other important number relationships, and concern for double-checking results.
- *Flexibility* requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible to be able to choose an appropriate strategy for the problem at hand and also to use one method to solve a problem and another method to double-check the results.

Facts and Fluency

- Conceptual subitizing
- Commutativity and associativity
- Doubles and n + 1
- Fives and tens frames
- Break-Apart-to-Make-Ten

How Can We Help Students with Facts?

- Ongoing practice and engagement with math facts tasks
- Hands-on activities and thoughtful discussions
- Conceptual understanding of operations
- Strategic thinking

Conceptual Understanding

- Understanding operations
  - Symbolic representations
  - Relationship between parts and whole
- Understanding is gained through:
  - Problem posing
  - Hands-on exploration
  - Classroom discussions
  - Real-world examples

Strategic Thinking

- Math fact strategies
  - Focuses attention on number sense, operations, patterns, properties, number concepts
  - Big ideas: concept of tens, knowing the order of addends will not affect the sum, various numbers can create the same sum (e.g., 5 + 4 = 9 and 6 + 3 = 9), and that there is a unique relationship between those two equations (the first addend is one more and the second is one less in the second number sentence than the first)

Meaningful Practice

Builds on understanding of operations and using strategic reasoning to explore math facts

- Practice 5 – 10 minutes daily throughout the school year
- Vary the practice activities - ensures that students are motivated and engaged
- Automaticity is achieved through brief, frequent, interactive activities

Homework

- Read pages 1-29 in *Mastering the Basic Math Facts in Addition and Subtraction*.
- Be ready to discuss the questions on the homework sheet.